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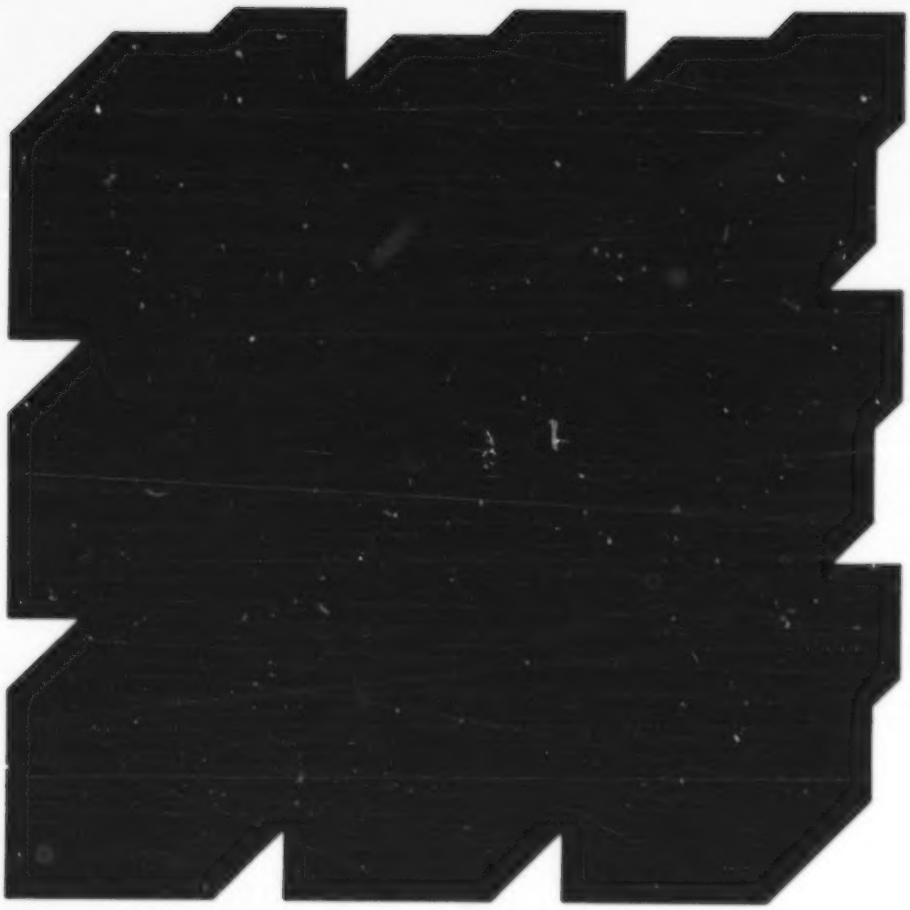

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DISCUSSION PAPER NO. 288

Savings and Taxes in a Life Cycle Growth Model with Age-Earnings Profile

by

Denis Gauthier

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RÉSUMÉ

Dans le présent document, les taux d'épargne et leurs élasticités par rapport au taux d'intérêt sont calculés à l'aide d'un modèle du cycle de vie à période multiple. L'auteur démontre que les hypothèses concernant la structure âge-temps des revenus du travail d'un particulier et leurs effets sur la contrainte budgétaire relative à l'ensemble de sa durée de vie peuvent avoir sur les taux d'épargne et sur leurs élasticités par rapport au taux d'intérêt un effet aussi marqué que celui qu'exercent le choix de la forme fonctionnelle et la valeur des paramètres de la fonction d'utilité intertemporelle de l'individu. Une fois le modèle calibré afin de reproduire des taux d'épargne réalistes, les élasticités de l'épargne par rapport au taux d'intérêt deviennent inférieures à celles estimées par Summers [1981a]. Elles sont toutefois supérieures à l'unité.

Le modèle du cycle de vie est ensuite incorporé à un modèle néoclassique de croissance équilibrée et est utilisé pour évaluer les effets sur le bien-être de diverses options de rechange à l'imposition du revenu du capital dans un contexte d'équilibre général. Les résultats indiquent qu'un impôt sur les traitements et salaires serait légèrement préférable à un système d'impôt sur le revenu, et que les particuliers verraient leur situation s'améliorer encore davantage sous un régime d'impôt à la consommation. L'amélioration du bien-être sous un régime d'impôt à la consommation dans une situation d'équilibre de croissance pourrait représenter jusqu'à 10 % du revenu que touche le particulier durant sa vie.

L'auteur procède en outre à la dérivation d'un ensemble d'impôts optimaux. Prenant comme point de comparaison l'impôt sur le revenu payé dans le scénario de référence, il en vient à la conclusion que l'amélioration du bien-être sous un régime d'impôt optimal pourrait jusqu'à doubler celle qu'on obtiendrait sous le régime d'un impôt à la consommation.

L'auteur, en conclusion, avertit le lecteur que ces résultats ont été obtenus à l'aide d'un modèle d'économie fermée, et que les conséquences exercées sur le bien-être par un régime d'impôt sur les traitements et salaires ou un régime d'impôt à la consommation seraient sensiblement moindres dans un contexte d'économie ouverte.

ABSTRACT

In this paper, saving rates and their interest rate elasticities are estimated using a multi-period life cycle model. It is shown that assumptions concerning the age-time profile of individual earnings and their effects on the lifetime budget constraint can have as much impact on saving rates and their interest rate elasticities as the choice of functional form and parameter values of the individual's lifetime utility function. The calibration of the model to yield realistic levels of savings produces interest rate elasticities of saving lower than those evaluated by Summers [1981a], but they are still in excess of unity.

The life cycle model is then incorporated into a neoclassical steady-state growth model and used to evaluate the general equilibrium welfare consequences of alternatives to the taxation of capital income. The results show that a wage tax would be slightly preferred to an income tax system and that individuals would fare even better under a consumption tax. The welfare gains of consumption taxation in steady-state growth equilibrium could be as much as 10 per cent of lifetime income for the representative individual.

A set of optimal taxes are also derived. With reference to the income tax base case, the welfare gains attainable under the optimal tax system could be as much as twice those obtained with a consumption tax.

The paper ends with the warning that these results were obtained in a closed economy model and that the welfare consequences of wage taxation or consumption taxation would be considerably smaller in an open economy context.

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1 INTRODUCTION

A continuing debate among public finance economists concerns the choice of an appropriate tax base. Should individuals be taxed on the basis of their contributions to the economy as measured by their incomes; or, alternatively, should they be taxed on the basis of their consumption or what they "take out" of the economy? Furthermore, if the income base were adopted, should capital income be included or not in the tax base?

In an influential article, Summers [1981a] provided a numerical analysis of these issues in the context of a general equilibrium model. His simulations of the economy under different tax bases lead him to conclude that "the welfare cost of capital income taxation may have been seriously underestimated. [...], the annual welfare gain from a shift to consumption taxation is conservatively estimated at 10 per cent of GNP." (Summers [1981a, p. 543])

In deriving his results, Summers also made advances that affect public policy in other areas -- for instance, he found a high interest rate elasticity of savings where the conventional wisdom was previously that this elasticity was very small if at all different from zero.

Summers' work has therefore attracted close examination, criticisms and modifications by other economists such as Evans [1983], Starrett [1983], and Auerbach, Kotlikoff and Skinner [1983].

In this paper, we propose to re-examine the issues concerning the interest rate elasticity of savings and the welfare implications of alternatives to the taxation of capital income under more realistic conditions. Following Summers [1981a], the analysis is carried out in a differential incidence framework and relies on a multi-period single-sector neoclassical steady-state growth model with continuously overlapping generations of life cycle savers. However, our study departs from Summers' examination of the issues in the following respect. Instead of assuming that at any instant workers of different ages have the same productivity and therefore receive the same wage, we adopt the more realistic assumption of a hump-shaped instantaneous age-earnings profile. Since saving is defined as the difference between after-tax income and gross-of-tax consumption, we feel that as much effort should be devoted to the specification of the age-time structure of individual earnings and its effect on the lifetime budget constraint as to the choice of the functional form and parameter values for the lifetime utility function.

This paper also offers an innovation in the debate on the choice of a tax base. In addition to conventional alternatives to the taxation of capital income such as wage taxation and consumption

taxation, we derive a set of optimal taxes and provide a general equilibrium numerical analysis of their welfare implications.

Section II of the paper describes the demand side of the model. A multi-period overlapping generations life cycle model is developed. Assumptions concerning the age-time profile of earnings are presented. Aggregate saving and its interest rate elasticity are estimated, and sensitivity analysis is performed on the estimates. This section ends with a calibration of the demand side of the model to yield realistic levels of savings. A single sector production side is added to the model in Section III. Section IV provides steady-state comparisons of tax alternatives in a closed economy. General equilibrium solutions of the model for an income tax system and for wage and consumption tax systems yielding equal tax revenues are presented. The welfare consequences of the alternative tax systems are analyzed using a wealth equivalent measure of the welfare cost of capital income taxation. The model is optimized in Section V. General equilibrium solutions with optimal taxes and their welfare implications are presented in this section. Section VI concludes the paper by reviewing the main findings and suggesting an extension to the model.

2 THE CONSUMPTION/SAVINGS CHOICE IN A LIFE CYCLE MODEL WITH OVERLAPPING GENERATIONS

At a given point in time, the demand side of the model consists of N individuals divided up into a series of continuously overlapping generations according to the following age density function:[1]

$$(1) \qquad f(v) = \frac{ne^{-nv}}{1-e^{-nV}},$$

where n = rate of population growth, and

V = certain age of death.

Individual preferences for consumption in different periods are governed by an additive, isoelastic utility function. The discounted value of a representative individual's lifetime utility is:

$$(2) \qquad U = \int_0^T \frac{C(t)^{\gamma}}{\gamma} e^{-\delta t} dt,$$

where $C(t)$ = consumption at age t,

$1-\gamma$ = elasticity of the marginal utility function,[2] and

δ = utility discount rate.

Individual savings or asset accumulation is given by the following differential equation:

$$(3) \qquad \dot{a}(t) = r(1-t_r)a(t) + (1-t_w)W(t) - \frac{C(t)}{1-t_c},$$

where $a(t)$ = individual assets at age t,

$\dot{a}(t)$ = change in assets or saving at age t,

$W(t)$ = wage at age t,

r = constant gross of tax rate of return on capital,

t_r = capital income tax rate,

t_w = wage tax rate, and

t_c = consumption tax rate.

The optimization problem of the representative individual is to maximize expression (2) subject to the equation of motion (3). Following the maximum principle technique, we can write the Hamiltonian function:

$$(4) \qquad H = \frac{C(t)^{\gamma}}{\gamma} e^{-\delta t} + \lambda(t)[r(1-t_r)a(t) + (1-t_w)W(t) - \frac{C(t)}{1-t_c}].$$

The necessary conditions are:

$$(5) \qquad \frac{\partial H}{\partial C(t)} = C(t)^{\gamma-1} e^{-\delta t} - \frac{\lambda(t)}{1-t_c} = 0,$$

$$(6) \qquad \frac{\partial H}{\partial a(t)} = \lambda(t)r(1-t_r) = -\dot{\lambda}(t), \text{ and}$$

$$(7) \qquad \frac{\partial H}{\partial \lambda(t)} = r(1-t_r)a(t) + (1-t_w)W(t) - \frac{C(t)}{1-t_c} = \dot{a}(t).$$

The canonical equation (6) for the costate variable implies:

$$(8) \qquad \lambda(t) = \lambda(0)e^{-r(1-t_r)t}.$$

Substituting this expression into equation (5) and taking the time derivative of the latter, we can derive the rate of growth of consumption along the optimal consumption path:

$$(9) \qquad \frac{\dot{C}(t)}{C(t)} = \frac{r(1-t_r)-\delta}{1-\gamma}.$$

The solution to equation (9) is:

$$(10) \qquad C(t) = C(0)e^{[r(1-t_r)-\delta]t/(1-\gamma)}.$$

Initial consumption C(0) can be found by solving equation (7). The solution to the equation of motion is:

$$(11) \qquad a(t) = e^{r(1-t_r)t}\{a(0) + \int_0^t [(1-t_w)W(u) - \frac{C(u)}{1-t_c}]e^{-r(1-t_r)u}\,du\}.$$

It is assumed in this model that there are no bequests,[3] hence a(0)=a(T)=0. This implies:

$$(12) \quad \int_0^T [(1-t_w)W(t) - \frac{C(t)}{1-t_c}]e^{-r(1-t_r)t}\,dt = 0.$$

Substituting equation (10) into equation (12), and solving for C(0) yields:

$$(13) \quad C(0) = \frac{(1-t_c)(1-t_w)\{[\gamma r(1-t_r)-\delta]/(1-\gamma)\}}{e^{[\gamma r(1-t_r)-\delta]T/(1-\gamma)}-1} H,$$

$$\text{where } H = \int_0^{T'} W(t)e^{-r(1-t_r)t}\,dt.$$

Initial consumption is a fraction of lifetime income H, which itself depends on the time profile of labour income up to the age of retirement T'. In our model, two factors account for the evolution of the representative individual's labour income through time. One factor is age-specific. Its effects depend on the instantaneous age-earnings structure. The other factor is time-specific and stems from aggregate labour productivity growth. We make the following assumptions concerning these two factors.

a) <u>Instantaneous Age-Earnings Structure Assumption</u>

Since we are interested in the consumption behaviour of individuals at different stages of their lives, it is assumed that individuals of the same age are homogeneous with respect to their labour services and hence receive the same wage. However, because

of differences in education, training, experience, etc., workers in different age groups have different productivities and, accordingly, receive different wages following a hump-shaped age-earnings profile given by:

$$(14) \quad W(v) = W(0)e^{(bv+cv^2)}.$$

Values for b and c can be set by specifying the age group M that earns the maximum wage and the ratio of earnings between any two age groups (j, k).

Thus,

$$(15) \quad b = -2M \ln[W(j)/W(k)]/[(j-k)(j+k-2M)], \text{ and}$$

$$(16) \quad c = \ln[W(j)/W(k)][(j-k)(j+k-2M)].$$

The average wage in any given period can be calculated as follows:

$$(17) \quad \bar{W} = \frac{N}{L} \int_0^{V'} f(v)\, W(v)\, dv,$$

where $L = \frac{(1-e^{-nV'})}{1-e^{-nV}} N$ is the number of individuals in the work force. After substitution of expressions (1) and (14) into (17), the average wage can be expressed as:

(18) $\bar{W} = \Omega\, W(0)$,

where $\Omega = \frac{n}{(1-e^{-nV'})} \int_0^{V'} e^{(b-n+cv)v} dv$.

The wage of a person just starting his or her working life is a fraction of the average wage in the economy; that fraction being a function of the rate of growth of population, the age of retirement and the shape of the instantaneous age-earnings structure.

Equation (14) can now be written as:

(19) $W(v) = \frac{\bar{W}}{\Omega} e^{(bv+cv^2)}$.

b) Aggregate Labour Productivity Growth Assumption

Due to aggregate labour productivity increases, the average wage grows exponentially by g per cent per period.

(20) $\bar{W}(t) = \bar{W}(0)e^{gt}$.

The following implications can be derived from the above assumptions. Equation (20) describes the time trend of the average wage based on aggregate labour productivity growth, while equation (19) represents age-specific labour productivity deviations around that trend. Combining equations (19) and (20) yields the individual wage-time profile.

$$(21) \quad W(t) = \frac{\bar{W}(0)}{\Omega} e^{(g+b+ct)t}.$$

c) Characteristics of the Age-Time Profile of Earnings

i) The instantaneous age-earnings structure being independent of time, the relative distribution of earnings among age groups does not change over time.

ii) The average wage grows at g per cent per period.

iii) The individual rate of earnings growth is:

$$(22) \quad \frac{\dot{W}}{W} = g + b + 2ct.$$

Since $b+2ct \gtrless 0$ as $t \lessgtr M$, $(\dot{W}/W) \gtrless g$ as $t \lessgtr M$. The productivity of individuals increases by more than the average until they reach age M. After that age, they experience a slower rate of productivity improvement than the average.

iv) The rate of growth of earnings for the individual declines through time.

$$(23) \quad \frac{d(\dot{W}/W)}{dt} = 2c < 0.$$

However, for plausible values of b, c, and g, the point at which earnings would start to decline may be outside the relevant range (i.e., larger than T', the age of retirement).

The age-time structure of earnings is illustrated in Figure 1. The instantaneous age-earnings profiles, along the v axis, show the usual hump shape. They are moving upward through time, along the t axis, according to the aggregate labour productivity assumption. The individual wage-time profile can be read on the wage surface above the 45° line dividing the (v, t) plane.

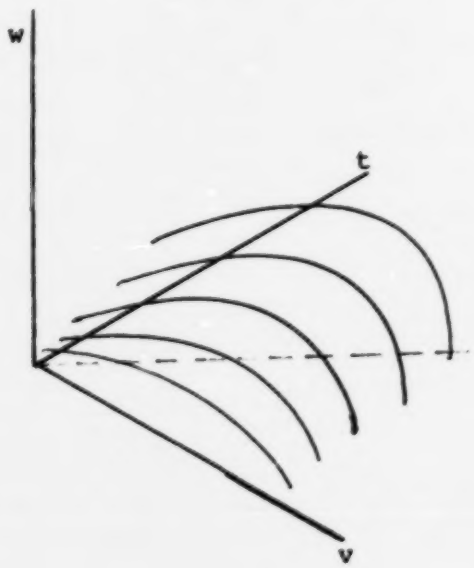

Figure 1

d) Solution for Aggregate Saving and Its Elasticity

We can now proceed to derive solutions for the paths of individual consumption and asset accumulation. Using equation (21), the expression for lifetime income becomes:

$$(24) \quad H = \frac{\bar{W}(0)\ \Psi}{\Omega},$$

$$\text{where} \quad \Psi = \int_0^{T'} e^{[g+b-r(1-t_r)+ct]t}\, dt.$$

Appropriate substitution of expressions (13) and (24) into equation (10) yields the individual consumption path:

$$(25) \quad C(t) = \frac{(1-t_c)(1-t_w)\{[\gamma r(1-t_r)-\delta]/(1-\gamma)\}\bar{W}(0)\Psi e^{[r(1-t_r)-\delta]t/(1-\gamma)}}{(e^{[\gamma r(1-t_r)-\delta]T/(1-\gamma)}-1)\,\Omega}.$$

Using the above equation and expression (21), equation (11) for individual asset accumulation can be rewritten as:

$$(26) \quad a(t) = (1-t_w)\frac{\bar{W}(0)}{\Omega}e^{r(1-t_r)t}\left\{\int_0^{t^*} e^{[g+b-r(1-t_r)+cu]u}\, du - \frac{\Psi(e^{[\gamma r(1-t_r)-\delta]t/(1-\gamma)}-1)}{e^{[\gamma r(1-t_r)-\delta]T/(1-\gamma)}-1}\right\},$$

$$\text{where } t^* = t \quad \text{for} \quad 0 \leq t \leq T', \text{ and}$$
$$t^* = T' \quad \text{for} \quad t > T'.$$

Aggregate consumption and asset accumulation at one point in time can be obtained by integrating equations (25) and (26) over the entire population.

$$(27) \quad C = N \int_0^V f(v)C(v)\,dv, \text{ and}$$

$$(28) \quad A = N \int_0^V f(v)a(v)\,dv.$$

Noting that the initial period average wage for an individual of age v is simply the current period average wage discounted back v periods at the average rate of growth of labour productivity, i.e. $\bar{W}(0) = \bar{W}e^{-gv}$, aggregate consumption is:

$$(29) \quad C = \frac{n(1-t_c)(1-t_w)\bar{W}L\Psi\{[\gamma r(1-t_r)-\delta]/(1-\gamma)\}(e^{\{[r(1-t_r)-\delta]/(1-\gamma)-n-g\}V}-1)}{(1-e^{-nV'})\Omega(e^{[\gamma r(1-t_r)-\delta]V/(1-\gamma)}-1)\{[r(1-t_r)-\delta]/(1-\gamma)-n-g\}},$$

and aggregate asset accumulation is:

$$(30) \quad A = \frac{\bar{W}(1-t_w)L}{[r(1-t_r)-n-g]}\left\{\frac{n\Psi\{[\gamma r(1-t_r)-\delta]/(1-\gamma)\}(e^{\{[r(1-t_r)-\delta]/(1-\gamma)-n-g\}V}-1)}{(1-e^{-nV'})\Omega(e^{[\gamma r(1-t_r)-\delta]V/(1-\gamma)}-1)\{[r(1-t_r)-\delta]/(1-\gamma)-n-g\}}-1\right\}.$$

Aggregate saving can be derived from the following definition:

$$(31) \quad S = \bar{W}(1-t_w)L + r(1-t_r)A - C/(1-t_c).$$

Substituting equations (29) and (30) into (31), we obtain the aggregate saving function:

$$(32)\quad S = \frac{(n+g)\bar{W}(1-t_w)L}{[r(1-t_r)-n-g]}\left\{\frac{n\Psi[\gamma r(1-t_r)-\delta]/(1-\gamma)(e^{\{[r(1-t_r)-\delta]/(1-\gamma)-n-g\}V}-1)}{(1-e^{-nV'})\Omega(e^{[\gamma r(1-t_r)-\delta]V/(1-\gamma)}-1)\{[r(1-t_r)-\delta]/(1-\gamma)-n-g\}}-1\right\}.$$

The corresponding long-run elasticity of saving with respect to the net-of-tax interest rate is:

$$(33)\quad \eta = \frac{r(1-t_r)n}{n\Psi BG-(1-e^{-nV'})\Omega DF}\left\{\frac{\Psi}{1-\gamma}[BG(V-\frac{1}{F}-\frac{\gamma V(D+1)}{D})+BV+G\gamma]+BG\phi\right\}$$

$$-\frac{r(1-t_r)}{[r(1-t_r)-n-g]},$$

where $B = [\gamma r(1-t_r) - \delta]/(1-\gamma)$,

$D = e^{BV} - 1$,

$F = [r(1-t_r) - \delta]/(1-\gamma)-n-g$,

$G = e^{FV} - 1$, and

$$\phi = \frac{\partial\Psi}{\partial r(1-t_r)} = \{[g+b-r(1-t_r)]\Psi-e^{[g+b-r(1-t_r)+cV']V'} + 1\}/2c.$$

If there were no instantaneous age-earnings profile, b=c=0, Ω=1 and equation (21) becomes $W(t)=\bar{W}(0)e^{gt}$. In any time period, all individuals earn the same wage and this wage grows exponentially through time at the rate g. In such circumstances, the above two equations become identical to those derived by Summers [1981a].[4]

Similarly, if g=0, equations (32) and (33) are representative of saving and saving elasticity functions derived from models in which the individual time path of wages is independent of technical change and is a function only of the instantaneous age-earnings profile.[5]

Consequently, equations (32) and (33) represent versions of the zero bequest, fixed labour supply, overlapping generations life cycle model, generalized by inclusion of both age and time factors influencing the individual time path of earnings.

In the life cycle model, savings arise because the time pattern of an individual's income does not coincide with his or her desired consumption; although the two must be equal in present value terms over the individual's lifetime. As shown in the next section, it turns out that assumptions concerning the age-time profile of individual earnings can have as much impact on the level of estimated aggregate saving and its interest rate elasticity as the choice of parameter values entering the utility function.

e) Saving Rates and Elasticities

Table 1 shows aggregate saving rates, saving elasticities and asset accumulation for the same set of parameter values appearing in Summers' Table 1 [1981a]. In constructing this table, it

Table 1

Aggregate Saving Rates, Saving Elasticities, and Asset Accumulation

	Value of After-Tax Interest Rate								
	.04			.06			.08		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
γ=.5 ε=2									
$S/\overline{W}(1-t_w)L$	.131	.093	.087	.450	.265	.393	.962	.573	.866
η	4.05	2.87	5.97	2.61	2.52	2.81	2.78	2.91	2.82
$A/\overline{W}(1-t_w)L$	3.75	6.21	2.49	12.86	17.69	11.22	27.48	38.19	24.75
γ=0 ε=1									
$S/\overline{W}(1-t_w)L$	.062	.059	.018	.187	.123	.137	.349	.215	.287
η	3.70	1.90	12.20	2.23	1.83	2.84	2.20	2.09	2.43
$A/\overline{W}(1-t_w)L$	1.77	3.92	.52	5.33	8.20	3.92	9.97	14.31	8.20
γ=-.5 ε=.66									
$S/\overline{W}(1-t_w)L$	.040	.048	-.004	.111	.084	.064	.194	.128	.140
η	3.44	1.35	36.07	2.04	1.41	3.30	1.88	1.59	2.35
$A/\overline{W}(1-t_w)L$	1.14	3.20	-.10	3.18	5.58	1.84	5.53	8.57	4.01
γ=-1 ε=.5									
$S/\overline{W}(1-t_w)L$	.029	.043	-.014	.076	.066	.031	.128	.093	.078
η	3.20	1.01	6.18	1.93	1.11	4.50	1.71	1.28	2.55
$A/\overline{W}(1-t_w)L$	.83	2.85	-.41	2.18	4.39	.88	3.66	6.19	2.24
γ=-2 ε=.33									
$S/\overline{W}(1-t_w)L$	.018	.038	-.025	.044	.049	-.001	.070	.062	.024
η	2.72	.58	1.88	1.79	.74	69.13	1.56	.88	4.26
$A/\overline{W}(1-t_w)L$	.52	2.51	-.72	1.25	3.28	-.03	2.01	4.13	.68
γ=-5 ε=.166									
$S/\overline{W}(1-t_w)L$	.008	.033	-.035	.013	.034	-.031	.020	.035	-.024
η	1.08	.038	.18	1.43	.13	.58	1.52	.24	1.29
$A/\overline{W}(1-t_w)L$	.22	2.17	-1.01	.37	2.24	-.88	.57	2.36	-.68

Note: Numbers in the first column assume no instantaneous age-earnings profile (i.e., b=c=0), but allow for productivity growth. Numbers in the second column are based on calculations assuming zero productivity growth and an instantaneous age-earnings structure where people in their 30th working year have the highest earnings; their earnings being 50 per cent higher than earnings of people just starting their working life. Numbers in the third column are calculated on the basis of the instantaneous age-earnings structure assumption of column (2) and the productivity growth assumption of column (1). ε is the consumption intertemporal elasticity of substitution.

Assumptions: V=50, V'=40, δ=.03, n=.015, g=.02 (where applicable).

was assumed that individuals live 50-year economic lives with retirement at "age" 40, that population grows at 1.5 per cent per annum, and that utility is discounted at 3 per cent. Where applicable, aggregate labour productivity is assumed to grow by 2 per cent per year and the hump-shaped instantaneous age-earnings profile is characterized by individuals in their 30th working year having the highest earnings of any age group; their earnings being 50 per cent higher than earnings of individuals who are just starting their working life.[6]

Three series of figures are presented in Table 1 for each pair of values of the after-tax interest rate and the intertemporal elasticity of substitution in consumption. The figures in column (1) assume a flat instantaneous age-earnings profile (i.e., b=c=0) but allow for aggregate labour productivity growth of 2 per cent per annum. These are Summers' [1981a] assumptions. However, the saving rates and elasticities appearing under column (1) are different from those in Summers' Table 1. It is this author's impression that Summers was a victim of a computational error when calculating figures for his Table 1.[7] Summers [1981a, p. 537] made a "prima facie theoretical case for a high interest elasticity of savings." It turns out that, using his parametric assumptions, the correctly calculated saving elasticities are consistently higher than those published in his AER article by an absolute value of .34.

The figures in the second column are based on calculations assuming no technological change and the hump-shaped instantaneous

age-earnings profile described above. In all but one case, the saving elasticities in column (2) are lower than those in column (1).

Figures in the third column are based on the more realistic assumptions of both the hump-shaped instantaneous age-earnings profile and aggregate labour productivity growth of 2 per cent per annum. The resulting saving rates are all lower than those in column (1); some are even negative, while the saving elasticities are higher than those derived under Summers' assumptions except for the last row of the table and one case in the second-last row.

The main picture emerging from Table 1 is that the saving rates and saving elasticities vary widely depending on the parametric assumptions concerning the age-time profile of earnings and the intertemporal elasticity of substitution with regard to consumption. For example, at an after-tax interest rate of .06, the calculated saving rate varies from -.031 to .450,[8] and the saving elasticity ranges from .13 to 69.13.[9]

Since the choice of an efficient tax base to be discussed later depends critically on the size of the saving elasticity, values for parameters entering the utility function and the derived aggregate saving function must be selected carefully. For the rest of this paper, we use the saving function derived under the assumptions of the hump-shaped instantaneous age-earnings

profile described earlier and an annual growth rate of 2 per cent in aggregate labour productivity. It also seems reasonable to assume an economic life lasting 50 years with retirement at age 40, and population growth of 1.5 per cent per annum.

More difficult choices are involved in the selection of values for the intertemporal elasticity of substitution in consumption and the utility discount rate. Regarding the former, the literature suggests a wide range of estimates. Weber [1970, 1975] found values ranging from .13 to .75. Friend and Blume [1975] reported an elasticity of less than .5; Ghez and Becker [1975] estimated it to be at most .28; Grossman and Shiller [1981] proposed less than .25; Hall [1981] found values smaller than .1, and Summers [1981b] found values from .04 to .45. The above estimates range from almost 0 to .75. There is no empirical evidence regarding the time preference rate.[10] Following Evans [1983], we will use utility discount rates ranging from -3 to 3 per cent.

The type of sensitivity analysis presented in Table 1 using different pairs of intertemporal elasticities of substitution in consumption and utility discount rates could be extended but many of the results would be implausible. Instead, it seems more appropriate to calibrate the demand side of the model to a particular benchmark solution. The benchmark case considered is given by an aggregate long run saving rate out of disposable labour income equal to 20 per cent when the after-tax rate of interest is 5 per cent.[11]

Table 2 presents aggregate saving rates, saving elasticities and asset/net labour income ratios for various values of the after-tax interest rate assuming pairs of ε and δ that yield the benchmark case. The range of calculated saving elasticities is still considerable, varying from .55 to 6.22. Our preferred set of parameter values which we shall use in subsequent simulations are $\delta=0$ and $\varepsilon=.73$. Although theoretically conceivable, negative utility discount rates were ruled out as being intuitively unappealing.[12] Of the cases with non-negative discount rates, the zero utility discount rate with $\varepsilon=.73$ is preferred for two reasons. First, it is the only case with an intertemporal elasticity of substitution in consumption within the range of empirical estimates mentioned earlier. Secondly, and perhaps more importantly, the three cases with positive utility discount rates yield dissaving or negative net worth over several periods in youth.[13]

f) Summary

To summarize thus far, the introduction of a more realistic hump-shaped instantaneous age-earnings profile and the calibration of the demand side of the model to yield more plausible saving rates produce long run interest rate elasticities of saving that are lower than those estimated by Summers. They are still in excess of unity, ranging from 1.1 at an after-tax interest rate of 4 per cent to 1.5 at an after-tax interest rate of 8 per cent. It remains to be seen what these lower saving elasticities imply in terms of steady-state welfare change when considering a wage tax

Table 2

Aggregate Saving Rates, Saving Elasticities, and Asset Accumulation

(δ and γ chosen to yield $S/\bar{W}(1-t_w)L = .2$ at $r(1-t_r) = .05$)

	Value of after-tax interest rate				
	.04	.05	.06	.07	.08
$\delta = .03$ $\gamma = .45$ $\varepsilon = 1.83$					
$S/\bar{W}(1-t_w)L$	.075	.200	.347	.528	.761
η	6.22	3.35	2.80	2.70	2.79
$A/\bar{W}(1-t_w)L$	2.15	5.71	9.91	15.09	21.75
$\delta = .02$ $\gamma = .18$ $\varepsilon = 1.22$					
$S/\bar{W}(1-t_w)L$	.119	.200	.293	.403	.538
η	2.59	2.15	2.06	2.11	2.25
$A/\bar{W}(1-t_w)L$	3.40	5.71	8.36	11.51	15.38
$\delta = .01$ $\gamma = -.09$ $\varepsilon = .92$					
$S/\bar{W}(1-t_w)L$	.141	.200	.266	.343	.434
η	1.58	1.55	1.60	1.70	1.84
$A/\bar{W}(1-t_w)L$	4.03	5.71	7.61	9.80	12.39
$\delta = 0$ $\gamma = -.37$ $\varepsilon = .73$					
$S/\bar{W}(1-t_w)L$	.155	.200	.251	.308	.374
η	1.12	1.19	1.28	1.40	1.53
$A/\bar{W}(1-t_w)L$	4.42	5.71	7.16	8.80	10.69
$\delta = -.01$ $\gamma = -.64$ $\varepsilon = .61$					
$S/\bar{W}(1-t_w)L$	.164	.200	.240	.285	.336
η	.85	.95	1.06	1.17	1.30
$A/\bar{W}(1-t_w)L$	4.68	5.71	6.86	8.15	9.61
$\delta = -.02$ $\gamma = -.91$ $\varepsilon = .52$					
$S/\bar{W}(1-t_w)L$	.170	.200	.233	.269	.310
η	.67	.78	.89	1.00	1.11
$A/\bar{W}(1-t_w)L$	4.86	5.71	6.65	7.69	8.85
$\delta = -.03$ $\gamma = -1.18$ $\varepsilon = .46$					
$S/\bar{W}(1-t_w)L$	.175	.200	.227	.257	.291
η	.55	.65	.75	.86	.96
$A/\bar{W}(1-t_w)L$	5.00	5.71	6.49	7.35	8.30

Note: The above results are based on calculations assuming an instantaneous age earnings structure where people in their 30th working year have the highest earnings; their earnings being 50% higher than earnings of people just starting their working life. ε is the intertemporal elasticity of substitution in consumption; δ is the utility discount rate.

Assumptions: $V = 50$, $V' = 40$, $n = .015$, $g = .02$

or a consumption tax as alternatives to an income tax. In order to answer the latter question, we first have to specify the supply side of the model, a task to which we now turn.

3 THE PRODUCTION SECTOR

The production side of the model is represented by an aggregate Harrod-neutral CES production function.

$$(34) \quad Y(t) = \{\alpha\, K(t)^{(\sigma-1)/\sigma} + (1-\alpha)\, [L(t)e^{gt}]^{(\sigma-1)/\sigma}\}^{\sigma/(\sigma-1)},$$

where $Y(t)$ = aggregate output at time t,

$K(t)$ = aggregate capital stock at time t,

$L(t)$ = labour force at time t,

$0<\alpha<1$ is the distribution parameter,

$0<\sigma<\infty$ is the elasticity of substitution between capital and labour, and

g = rate of growth of labour-augmenting technical progress.

The above expression describes output as being a homogenous function of two inputs: capital and labour. More correctly, in our model there is a continuous set of heterogenous age specific labour inputs. However, since age related productivity differences are already embedded in our model via equation (14) -- reflecting our assumption concerning the instantaneous age-earnings structure -- equation (34) is a valid production function specification where L is a composite labour input. Accordingly, a marginal unit of labour is here defined to be representative of all types of labour weighted by their respective proportions in the labour force as given by the age density

function. As a result, the marginal condition for efficient use of labour in the production process will yield the wage of the composite labour input, i.e., the average wage derived in expression (17).

The derived demands for capital and labour inputs are obtained from the following marginal conditions for cost minimization:

$$(35) \quad r(t) = \alpha \left\{\alpha+(1-\alpha) \left[K(t)/(L(t)e^{gt})\right]^{(1-\sigma)/\sigma}\right\}^{1/(\sigma-1)}, \text{ and}$$

$$(36) \quad \bar{W}(t) = (1-\alpha) \left\{\alpha\left[K(t)/(L(t)e^{gt})\right]^{(\sigma-1)/\sigma} + 1-\alpha\right\}^{1/(\sigma-1)} e^{gt}.$$

For r to be constant through time, steady-state growth implies that $[K(t)/L(t)e^{gt})]$ is also constant through time. Equation (36) then expresses the wage per unit of the composite labour factor as being equal to the wage per "efficiency" unit of composite labour growing through time at rate g as in equation (20). Since we are interested in steady-state equilibria, it is more convenient to analyze the system in period 0 for which the wage per unit of composite labour and the wage per "efficiency" unit of composite labour are equal. Deleting the time argument, equations (35) and (36) can be rewritten as:

$$(37) \quad r = \alpha \left[\alpha + (1-\alpha) (K/L)^{(1-\sigma)/\sigma}\right]^{1/(\sigma-1)}, \text{ and}$$

$$(38) \quad \bar{W} = (1-\alpha) \left[\alpha(K/L)^{(\sigma-1)/\sigma} + 1-\alpha\right]^{1/(\sigma-1)}.$$

Following Summers [1981a], the model will be solved with elasticities of substitution in production of 1 and .5. In both cases, the distribution parameter α is set so that labour's share of total output is .75 in the base case equilibrium with an income tax.[14]

4 STEADY-STATE COMPARISONS OF TAX ALTERNATIVES IN A CLOSED ECONOMY

a) General Equilibrium Solution

In the closed economy version of the model, the stock of productive capital is financed by net aggregate asset accumulation:[15]

(39) $K = A.$

Using the above equilibrium condition for the capital market, the model can be reduced to a system of three equations [equations (30), (37), and (38)] to be solved for r, $\bar{W}$, and K/L. The results can then be substituted in the appropriate equations to find the equilibrium level of consumption, savings and output. The method of solution is described in detail in the appendix.

The model is first solved for an income tax system with tax rates of 20 per cent on labour income and 50 per cent on capital income. Following Summers [1981a], the latter rate is assumed to represent the combined effects of corporate, property and individual capital income taxes. It is also assumed that the government runs a balanced budget. Having determined the level of government revenues raised by the income tax, the model is then solved for two equal-yield tax alternatives: a wage tax and a consumption tax. Steady-state equilibrium solutions of the model for the three tax systems are presented in Table 3.

Table 3

Steady-State Comparisons of Tax Alternatives (t_r = .5)

	Income Tax		Wage Tax		Consumption Tax	
	σ=1	σ=.5	σ=1	σ=.5	σ=1	σ=.5
r	.086	.086	.064	.057	.054	.044
$r(1-t_r)$	.043	.043	.064	.057	.054	.044
$\bar{W}$	1.069	2.032	1.181	2.300	1.253	2.438
$\bar{W}(1-t_w)$	.855	1.626	.789	1.555	1.253	2.438
Y/L	1.425	2.710	1.575	2.883	1.671	2.968
C/L	.889	1.690	.968	1.777	1.006	1.802
K/Y	2.893	2.893	3.905	3.578	4.663	4.051
S/Y	.101	.101	.137	.125	.163	.142
η	1.139	1.139	1.328	1.250	1.223	1.145
$\bar{W}L/Y$	.75	.75	.75	.798	.75	.821
t_w	.2	.2	.332	.324	0	0
t_r	.5	.5	0	0	0	0
t_c	0	0	0	0	.280	.293
α	.25	.723	.25	.723	.25	.723

Note: α is set so as to yield $\bar{W}L/Y$ = .75 in the income tax case.
σ is the elasticity of substitution in production between capital and labour.

Assumptions: V = 50, V' = 40, δ = 0, n = .015, g = .02, γ = -.366 (ε = .732). b and c are chosen such that people in their 30th working year have the highest earnings on a cross-section basis; their earnings being 50% higher than earnings of people just starting their working life.

In the base case with an income tax, the model yields a gross of tax rate of return on capital of 8.6 per cent implying a net rate of return to savers of 4.3 per cent and a saving/output ratio of 10.1 per cent. The replacement of the tax on capital income by either a wage or a consumption tax eliminates the wedge between the gross and the net of tax rates of return. The wage tax system always yields a rate of return on capital higher than that obtained under the consumption tax system.[16]

The higher net rates of return to savers under the two alternatives to an income tax system lead to higher levels of savings. The saving/output ratios increase to 13.7 per cent under the wage tax system and to 16.3 per cent under the consumption tax system when the production function is of the Cobb-Douglas type. With σ=.5, the increases in the saving/output ratios are smaller, going to 12.5 and 14.2 per cent under the wage and consumption tax systems, respectively.

The larger savings serve to finance a higher degree of capital intensity in the production process. Accompanying this capital deepening are higher outputs, lower rates of return on capital and higher gross wages. After-tax wages are, however, lower in the wage tax alternative due to the higher tax rates on labour income needed to offset the tax revenues lost with the elimination of the tax on capital income.

The extent of factor substitution in production does not alter the interpretation of results in comparing the alternative tax systems. Only relative magnitudes of changes in the variables are affected. With more factor substitution, the solutions for the two alternatives yield larger relative increases in the amount of capital and output, while the relative increases in wages are smaller than in the case with σ equal to .5. Also, while labour's share of output is fixed at 75 per cent under the three tax systems when $\sigma = 1$, an elasticity of substitution in production of .5 yields labour shares of .80 in the wage tax alternative and .82 in the consumption tax alternative.

According to the utility function (2), individuals essentially derive utility from the level and time pattern of consumption they can achieve throughout their lives. The results in Table 3 provide information on how the two alternative tax systems can affect the representative individual's timing and level of consumption. First, the higher after-tax rates of return to savers under the wage and consumption tax systems improve the intertemporal terms of trade by lowering the implicit price of future consumption. This effect includes Summers' human wealth effect and is welfare improving unless the representative individual incurs large debts in early years and, furthermore, has a somewhat low intertemporal elasticity of substitution with regard to consumption.[17] Second, the increased demand for labour accompanying the capital deepening leads to higher gross wages. This is equivalent to a positive gross wealth effect for the representative individual. The results in Table 3 indicate that

the intertemporal terms of trade effect would be stronger for the wage tax alternative, while the gross wealth effect would be larger for the consumption tax alternative. Finally, a negative net wealth effect results from the increase in either the wage tax rate or the consumption tax rate. The former reduces the disposable income of the representative individual during his or her working life, whereas the latter raises the gross price of consumption goods in every period.[18]

A wealth equivalent measure of the steady-state welfare consequences of the three tax alternatives for the representative individual is derived in the next section. Such a measure makes it possible to rank the tax alternatives in terms of economic efficiency.

b) Welfare Consequences of Alternative Tax Systems

Two different exercises regarding the welfare consequences of tax alternatives could be undertaken. One is to compare the effects of the three tax systems on the steady-state welfare of the representative individual. Another exercise would consist of evaluating the welfare gains or losses to all members of society of switching from an existing tax system to a different one. The latter exercise would take into account the transitional costs to all generations of such a change. We will deal here with the former question; the transitional issue being on our agenda for future research.

The question we are thus asking is the following one. Under which of the three equal yield tax systems in steady-state equilibria would the representative individual just starting his or her working life maximize his or her lifetime utility, and by how much? This question can be answered by constructing a wealth equivalent measure of the welfare cost of income taxation for the representative individual.

The indirect utility function obtained by substituting expression (25) into equation (2) is:

$$(40)\quad T=\frac{1}{\gamma}\left[\frac{(1-t_c)(1-t_w)\{[\gamma r(1-t_r)-\delta]/(1-\gamma)\}H}{e^{[\gamma r(1-t_r)-\delta]T/(1-\gamma)}-1}\right]^{\gamma}\int_0^T e^{[\gamma r(1-t_r)-\delta]t/(1-\gamma)}\,dt,$$

which upon integration gives:

$$(41)\quad T=\frac{1}{\gamma}\left[(1-t_c)(1-t_w)H\right]^{\gamma}\left[\frac{[\gamma r(1-t_r)-\delta]/(1-\gamma)}{e^{[\gamma r(1-t_r)-\delta]T/(1-\gamma)}-1}\right]^{\gamma-1}.$$

Using the above expression, the percentage change in human wealth needed in the income taxation base case for the representative individual to attain the utility level reached in the wage or consumption taxation system is:

$$(42)\quad \omega = 100\left\{ \ln\left[\frac{(1-t_{cj})(1-t_{wj})H_j}{(1-t_{ci})(1-t_{wi})H_i}\right] + \frac{(\gamma-1)}{\gamma}\ln\left[\frac{[\gamma r_j(1-t_{rj})-\delta]\,(e^{[\gamma r_i(1-t_{ri})-\delta]T/(1-\gamma)}-1)}{[\gamma r_i(1-t_{ri})-\delta]\,(e^{[\gamma r_j(1-t_{rj})-\delta]T/(1-\gamma)}-1)}\right]\right\},$$

where the subscript "i" refers to variables in the income tax base case, and the subscript "j" refers to variables in either the wage or consumption taxation case.

Evaluation of expression (42) using results from Table 3 indicates that the representative individual would prefer a consumption tax to either a wage tax or an income tax. The wage tax alternative is also slightly preferred to the income tax system. The welfare gains expressed as a percentage of lifetime income are 2.6 per cent in the wage tax case and 10.3 per cent in the consumption tax case when production follows Cobb-Douglas assumptions. With σ=.5, the welfare gains are smaller. They amount to 2 per cent of lifetime income in the wage tax case and 6.3 per cent in the consumption tax case. The superiority of consumption taxation is largely due to the fact that it leads to a higher capital intensity which in turn brings the system closer to the golden rule level of capital accumulation.

Recent work on the effects of investment tax credits, accelerated depreciation, inflation and other factors on the effective marginal rates of capital income taxation tend to show that the 50 per cent combined tax rate we have used may overestimate the true extent of capital income taxation. Accordingly, the model with the three equal yield tax alternatives was solved with a base case combined rate of capital income taxation of 35 per cent.[19] The results are presented in Table 4.

The ranking of the three tax systems in terms of economic efficiency remains unchanged. However, as there is a smaller intertemporal distortion in the income tax base case, the welfare gains associated with the wage and consumption tax alternatives are smaller. The consumption tax system produces welfare gains of 6.7 per cent with Cobb-Douglas technology, and 4.8 per cent when $\sigma=.5$. The wage tax system has little to offer in comparison with the income tax system. The gains in welfare are lower than 0.3 per cent of lifetime income.

Table 4

Steady-State Comparisons of Tax Alternatives (t_r = .35)

	Income Tax		Wage Tax		Consumption Tax	
	σ=1	σ=.5	σ=1	σ=.5	σ=1	σ=.5
r	.075	.075	.063	.058	.054	.047
$r(1-t_r)$	.049	.049	.063	.058	.054	.047
$\bar{w}$	1.120	3.361	1.189	3.636	1.253	3.852
$\bar{w}(1-t_w)$	.896	2.689	.835	2.572	1.253	3.852
Y/L	1.493	4.481	1.586	4.661	1.671	4.797
C/L	.965	2.894	1.010	2.979	1.043	3.024
K/Y	3.331	3.331	3.988	3.787	4.663	4.226
S/Y	.117	.117	.140	.133	.163	.148
η	1.181	1.181	1.313	1.265	1.223	1.164
$\bar{w}L/Y$	.75	.75	.75	.780	.75	.803
t_w	.2	.2	.298	.293	0	0
t_r	.35	.35	0	0	0	0
t_c	0	0	0	0	.254	.260
α	.25	.833	.25	.833	.25	.833
Welfare gain (as % of lifetime income)			.210	.295	6.656	4.786

Note: α is set so as to yield $\bar{w}L/Y$ = .75 in the income tax case.
σ is the elasticity of substitution in production between capital and labour.

Assumptions: V = 50, V' = 40, δ = 0, n = .015, g = .02, γ = -.366 (ε = .732), b and c are chosen such that people in their 30th working year have the highest earnings on a cross-section basis; their earnings being 50% higher than earnings of people just starting their working life.

5 OPTIMAL TAXATION

In the previous section of the paper, the welfare implications of three conventional tax systems were compared. This was done by first finding the Walrasian equilibria in our neoclassical growth model for three equal yield taxation systems with exogenously predetermined tax instruments. As is often the case in economics, market equilibria may not always be the most efficient outcomes. In the words of Samuelson [1958, p. 479], "... free pricing gets you on the Pareto-efficiency frontier but by itself has no tendency to get you to positions on the frontier that are ethically optimal in terms of a social welfare function; only by social collusions - of tax expenditure, fiat, or other type - can an ethical observer hope to end up where he wants to be."

The essence of the above message is that an additional instrument is needed to "optimize" the model. Without that instrument, the number of endogenous variables is just equal to the number of constraints (market equilibria and efficiency conditions). In these circumstances, the set of constraints can be "solved" to yield a general equilibrium solution to the model, but such a solution is not likely to maximize individual welfare. The same can be said about the wage tax and consumption tax versions of the model. Each version adds one instrument (the tax rate), and one constraint (the tax revenue requirements determined in the income tax case). Since the number of endogenous variables is still

equal to the number of constraints, the wage tax and consumption tax systems can be "solved", but again the solutions are not likely to maximize individual welfare. Only if two or more tax instruments are used will there be enough degrees of freedom (i.e., more endogenous variables than constraints) to "optimize" the model.

In this section, using the whole range of tax instruments in our model (wage tax, capital income tax, and consumption tax), we seek to answer the Samuelsonian [1959, p. 520] question: "What common rule or configuration would, if it were to be established for every representative man by universal (self-imposed) fiat, lead each representative man to the highest point on his preference field," given that a certain amount of tax revenues has to be collected?

This optimal tax mix can be derived by maximizing the representative individual's indirect utility function (41) subject to the capital, labour and goods markets being in equilibrium, the tax revenue requirements constraint, and the production efficiency conditions.

The following results are obtained:

(43) $r = n+g,$

$$(44) \quad t_r = 0,$$

$$(45) \quad t_w = 1 - \left\{ \left[\left(\frac{n+g}{\alpha} \right)^{\sigma} - n - g \right] \left[\frac{(D+1)V}{D} - \frac{1}{F} + \frac{\phi}{\Psi} \right] \right\}^{-1}, \text{ and}$$

$$(46) \quad t_c = 1 + \left[\frac{(D+1)V}{D} - \frac{1}{F} + \frac{\phi}{\Psi} \right] \left\{ \frac{1}{(1-\alpha)} \left[\left(\frac{n+g}{\alpha} \right)^{\sigma-1} - \alpha \right] \right\}^{\sigma/(\sigma-1)} \left\{ \frac{\Pi}{L} - (1-\alpha)^{\sigma/(\sigma-1)} \left[1 - \alpha \left(\frac{n+g}{\alpha} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \right\},$$

where Π is the tax revenue requirements determined in the income tax case, and D, F, ϕ, Ψ are evaluated for values of r and t_r given in expressions (43) and (44).[20]

Expression (43) implies that the optimized model yields the "golden rule" level of capital accumulation as the equilibrium rate of return on capital is equal to the rate of growth of output, n+g. Expression (44) indicates that capital income should not be taxed, nor subsidized, in the optimized version of our model. Unitary income elasticities of consumption and the simultaneous optimization of the wage and consumption taxes are necessary to obtain the latter result. If only one tax instrument could be used in addition to t_r, say t_w for example, the optimal t_r could be different from zero even with unitary expenditure elasticities.

The results for the optimal tax rates on wages and consumption are particularly interesting. The level of tax revenues to be raised (Π) does not appear in expression (45) for t_w, while it does show in expression (46) for t_c. Hence, the optimal wage tax rate is determined independently of tax revenue requirements. This is due to a special feature of our optimized model: the exact pairing of tax instruments with objectives. One objective is to have an equilibrium level of savings consistent with the golden rule level of capital accumulation. This objective is independent of the tax revenue requirements and is attained by setting t_w at the value for which it will produce the golden rule level of savings. The other objective is to collect a predetermined level of tax revenues. This objective is achieved by using the consumption tax which, because of the homotheticity of the utility function (2) and the implied unitary expenditure elasticities, is neutral with respect to savings.[21]

In the case where the no-tax equilibrium yields savings below the golden rule level, the optimal t_w will be negative. Savings will be increased to the golden rule level via wage subsidies. The tax on consumption will be used to finance the subsidy and to raise the predetermined level of tax revenues. On the other hand, if the no-tax equilibrium yields savings above the golden rule level, a tax on wages will reduce savings to their golden rule level. In the latter case, the tax on consumption may be either positive or negative depending on whether revenues collected under the wage tax fall short or not of the tax revenue requirements.

Steady-state equilibrium solutions of the model with optimal taxes are compared to the income tax base case solutions in Tables 5 and 6.

In both tables, the income tax base case solutions yield capital stocks well below their golden rule levels. The optimal tax solutions thus call for a large subsidization of wages. The subsidy is of the order of 150 per cent in the Cobb-Douglas case, and about one-third of that in the case where $\sigma=.5$. The accompanying taxes on consumption are such that in all cases they account for more than half of the gross-of-tax price of consumption goods.

Compared to the income tax system, the optimal tax mix described above involves intergenerational transfers which affect the steady-state general equilibrium of the system to produce the most efficient intertemporal allocation of consumption. That outcome could also have been achieved with other instruments had these been included in our model. For example, the intergenerational transfers could be obtained through a government budget surplus or else by establishing a social security scheme. The two latter measures could also be used to rectify the deficient level of savings in the economy.

A summary of the effects of all tax systems analyzed in this paper on the steady-state welfare of the representative individual

Table 5

Steady-State Equilibrium With Optimal Taxes
(t_r=.5 in income tax base case)

	Income Tax		Optimal Taxes	
	σ=1	σ=.5	σ=1	σ=.5
r	.086	.086	.035	.035
$r(1-t_r)$	.043	.043	.035	.035
$\bar{W}$	1.069	2.032	1.444	2.555
$\bar{W}(1-t_w)$	.855	1.626	3.605	3.620
Y/L	1.425	2.710	1.926	3.038
C/L	.889	1.690	1.053	1.810
K/Y	2.893	2.893	7.143	4.546
S/Y	.101	.101	.25	.159
$\bar{W}L/Y$	.75	.75	.75	.841
t_w	.2	.2	-1.496	-.417
t_r	.5	.5	0	0
t_c	0	0	.708	.500
α	.25	.723	.25	.723

Note: α is set so as to yield $\bar{W}L/Y$=.75 in the income tax case. σ is the elasticity of substitution in production between capital and labour.

Assumptions: V=50, V'=40, δ=0, n=.015, g=.02, γ=-.366 (ε=.732). b and c are chosen such that people in their 30th working year have the highest earnings on a cross-section basis; their earnings being which are 50% higher than earnings of people just starting their working life.

Table 6

Steady-State Equilibrium With Optimal Taxes
(t_r=.35 in income tax base case)

	Income Tax		Optimal Taxes	
	σ=1	σ=.5	σ=1	σ=.5
r	.075	.075	.035	.035
$r(1-t_r)$	.049	.049	.035	.035
$\bar{W}$	1.120	3.361	1.444	4.109
$\bar{W}(1-t_w)$	.896	2.689	3.605	6.334
Y/L	1.493	4.481	1.926	4.955
C/L	.965	2.894	1.090	3.045
K/Y	3.331	3.331	7.143	4.877
S/Y	.117	.117	.25	.171
$\bar{W}L/Y$	.75	.75	.75	.829
t_w	.2	.2	-1.496	-.541
t_r	.35	.35	0	0
t_c	0	0	.698	.519
α	.25	.833	.25	.833

Note: α is set so as to yield $\bar{W}L/Y$=.75 in the income tax case. σ is the elasticity of substitution in production between capital and labour.

Assumptions: V=50, V'=40, δ=0, n=.015, g=.02, γ=-.366 (ε=.732). b and c are chosen such that people in their 30th working year have the highest earnings on a cross-section basis; their earnings being 50% higher than earnings of people just starting their working life.

is presented in Table 7. The welfare changes are calculated with the income tax base case as the reference point. As expected, figures in the table reveal that the optimal tax system is more efficient than either the income tax, wage tax, or consumption tax alternatives in raising a given amount of tax revenues. The relative welfare gains made possible by the optimal tax system over and above those attained with the consumption tax are highest (lowest) for a combination of a high (low) elasticity of substitution in production and a low (high) tax rate on capital income in the income tax base case. For example, with σ=1 and t_r=.35 under the income tax base case, the welfare gain achieved with optimal taxes is twice as large as that attained with a consumption tax. On the other hand, with σ=.5 and t_r=.5 under the income tax base case, much of the potential welfare gain to the representative individual is already captured with a consumption tax that yields a 6.29 per cent gain compared to a gain of 7.34 per cent with optimal taxes.

The large welfare gains obtained with the optimal tax system are essentially due to the higher wages accompanying the large increase in capital intensity. Compared with the other tax alternatives, the intertemporal terms of trade are at their lowest since the optimal tax system yields the smallest net rate of return to savers. The low rate of return also contributes to increase the present value of taxes each individual pays during his/her lifetime.[22] The above two welfare worsening effects are, however, more than offset by a positive wealth effect taking the

Table 7

Summary of Steady-State Welfare Consequences of Alternative Tax Systems

	Welfare gain as % of lifetime income					
	Wage Tax		Consumption Tax		Optimal Tax	
	σ=1	σ=.5	σ=1	σ=.5	σ=1	σ=.5
t_r=.5 in the income tax base case	2.632	1.985	10.288	6.291	17.423	7.336
t_r=.35 in the income tax base case	.210	.295	6.656	4.786	13.634	6.501

Note: Figures in the table are based on results of Tables 3 to 6, and measure the steady-state welfare gains for the representative individual of being taxed according to the above tax systems as opposed to the income tax system defined in this paper. σ is the elasticity of substitution in production between capital and labour.

form of higher wages which reflect the increased marginal productivity of labour when capital expands to the golden rule level.

6 CONCLUSION

A multi-period overlapping generations life cycle model of savings behaviour augmented to incorporate a hump-shaped instantaneous age-earnings profile and aggregate labour productivity growth was developed. It was shown that the above two characteristics of the age-time structure of individual earnings had as much impact on estimated aggregate saving rates and their interest rate elasticities as the choice of parameter values entering the utility function. Sensitivity analysis on saving rates and elasticities was conducted using different pairs of intertemporal elasticities of substitution in consumption and utility discount rates. A set of values for these parameters was finally chosen by calibrating the demand side of the model to yield a particular benchmark saving rate. The implied long run interest rate elasticities of saving turned out to be lower than those evaluated by Summers [1981a], but they were still in excess of unity.

The neoclassical growth model was completed by adding a production sector behaving according to an aggregate Harrod-neutral CES production function.

General equilibrium solutions of the model were calculated in a differential incidence framework for an income tax system, a wage tax system, and a consumption tax system. These solutions were then used to estimate the steady-state welfare consequences of the

three tax systems for the representative individual. It was shown that an equal-yield wage tax system would be slightly preferred to an income tax system, while a consumption tax would yield even greater welfare gains to the representative individual.

Provided there were enough tax instruments, optimal taxes were derived. Within the confines of our simplified model which does not allow for government debt policy nor for social security programs, the intergenerational transfers needed to achieve the most efficient intertemporal allocation of consumption could be obtained by subsidization of wage earners and taxation of consumption. With reference to the income tax base case, the welfare gain attainable under the optimal tax system could be as much as twice that obtained with a consumption tax only.

The welfare gains obtained under the three alternatives to the income tax system we have analyzed were made possible to a great extent because of a high interest elasticity of savings that permitted a large expansion in the capital stock when taxes on capital income were removed. In the context of a small open economy where the domestic capital stock is determined by an exogenous world rate of return, the increase in savings would simply flow out of the country or displace foreign investment without affecting the level of capital in the economy. In the latter circumstances, the welfare consequences of wage taxation or

consumption taxation would be considerably smaller. In a separate paper [Gauthier, 1985], our model is extended to consider the implications of wage and consumption taxation in a small open economy.

APPENDIX

Because the functions Ω and Ψ do not have algebraic solutions and have to be approximated, we could not rely on any commercially available computer package to solve the model and had to write our own computer program to solve the system of non-linear equations. The steps involved in the fixed-point algorithm can be described as follows. First, Ω was approximated using Simpson's rule for approximate integration. For that purpose, the interval of integration was divided into 800 equal parts to which Simpson's formula was applied. Second, equations (37) and (38) were evaluated for a capital/labour ratio K1 equal to one. In the third step, Ψ was approximated over 800 equal parts using Simpson's rule and the value for r just derived in step 2. In the fourth step, Ω and the values for r, $\bar{W}$, and Ψ just derived were introduced on the right-hand side of equation (30). The resulting value was divided by L and compared to K1 which turned out to be a low guess value for the capital/labour ratio. Steps 2 to 4 were then repeated but with a guess K2 on the capital/labour ratio equal to 30. The value obtained in the evaluation of equation (30) proved that K2 was a high guess value for the capital/labour ratio.

K1 and K2 then constituted bounds on the equilibrium value of the capital/labour ratio. The Bolzano method of halving the intervals was then applied to find the fixed point. This

consisted in repeating steps 2 to 4 with an initial guess on the capital/labour ratio halfway between K1 and K2. At each iteration, K1 or K2 would change its value as the new low (high) guess value would replace K1(K2). The above process continued until the capital/labour ratio converged to its equilibrium value within a tolerance interval of .00001.

In the wage tax case, an additional calculation was added before proceeding to step 4. The tax rate on wages needed to meet the tax revenue requirements of the income tax base case was estimated. It could then be introduced with the other variables on the right-hand side of equation (30) in step 4.

In the consumption tax case, the tax rate was derived after the equilibrium values for r, $\bar{W}$, and K/L were obtained. This was done by first evaluating total output using the production function (34) and solving for t_c in equation (29) given that total output must be equal to C+S+Π.

All calculations in this paper were performed on a TRS-80 MC-10 computer using the Microsoft 1.0 version of BASIC.

Notes

1 Throughout this paper, the age variable v serves to differentiate age groups at one point in time; whereas the time variable t is used on a chronological basis to represent the age of a given individual. Both variables refer to the economic life of individuals, where age zero represents the start of their working life.

2 The intertemporal elasticity of substitution in consumption is simply $\varepsilon=1/(1-\gamma)$.

3 Evans [1983], and Starrett [1983] provide numerical analyses of saving rates and elasticities in a life cycle model with bequest motives. The effect of bequests on saving elasticities is very sensitive to the bequest specification in the utility function. If, following Barro [1974], bequests arise from the maximization of a utility function where the utility of future generations is an argument, then the long-run saving rate is infinitely interest elastic, as demonstrated earlier by Sidrauski [1967]. If, on the other hand, bequests are treated as own consumption in the final period, then small or even negative interest elasticities of savings are plausible.

4 This is so despite the fact that the utility functions are not identical. Summers' utility function is $U=\int_0^T C(t)^{\gamma}e^{-\delta t}dt$. The utility function used in this paper is proportional to the above function, the factor of proportionality being $1/\gamma$. Although both utility functions imply the same demand functions, utility function (2) is preferred on grounds of theoretical rigour since it respects the axiom of nonsatiation for any value of γ.

5 Auerbach, Kotlikoff, and Skinner [1983] use this type of model.

6 This implies the following parameter values for expression (14): b=.027031 and c=-.0004505.

7 Starrett [1983] also tried to replicate Summers' results with no success. Using Summers' assumptions, he derived values for the saving rates and saving elasticities identical to those in column (1) of Table 1.

8 The range of calculated saving rates is larger for higher interest rates.

9 The large magnitude of the 69.13 saving elasticity results simply from the fact that the corresponding saving rate is close to zero.

10 Although Olson and Bailey [1981] develop a case for a positive rate of time preference, they do so in the context of an infinite time horizon. With respect to a finite planning horizon they assert that "It is indeed fortunate that time preference within planning horizons of a lifetime or less in length is not the key issue [it may not be in their paper, but it is in our life cycle model], for it is almost impossible to find out what time preference over the life cycle might be." (p. 15)

11 Assuming a 20 per cent tax on wage income and a 75 per cent share of labour in total output, this implies a net saving rate out of total output of 12 per cent, and an aggregate asset/output ratio of 3.43.

12 A more realistic model could involve time varying utility discount rates. In such a model, some segments of the representative individual's lifetime could be characterized by negative rates of time preference. For example, when young, the individual would place a higher value on present consumption as opposed to consumption in the future. Later in life, the individual could attach more importance to the future as he or she becomes more concerned about family-related consumption or retirement consumption. As the remaining lifetime becomes shorter the individual would again prefer present over future consumption. This U-shaped pattern of utility discount rates would complicate the mathematical derivation of saving rates and elasticities. Professor Albert Ando pointed out that the calibration of the saving rate to a benchmark case would also become more difficult to perform as a new dimension is added to the problem. One would choose not only the lifetime average discount rate but also the degree of curvature of the U-shaped relationship between age and the utility discount rate. Given the current state of knowledge on utility discount rates, the latter would constitute a purely arbitrary choice.

13 Although a zero discount rate implies equal present valuation of "utils" originating at different moments in time, Böhm-Bawerk's first cause of preference for present over future consumption is still maintained given the diminishing marginal utility ($\gamma=-.37$) exhibited by the utility function. See Olson and Bailey [1981] for a discussion of Böhm-Bawerk's two principal causes of the preferences for present over future consumption.

14 Letting d denote labour's share of total output, $\alpha = \{1+[d/(1-d)](K^*/L^*)^{(\sigma-1)/\sigma}\}^{-1}$ where K^*/L^* is the equilibrium capital/labour ratio in the base case with an income tax. Although Summers made the same assumption concerning labour's share of output, it can be shown from his simulation results that he simply set α equal to .75 in the case where $\sigma=.5$. This produced a base case labour share of .71.

15 For simplicity, the model assumes no depreciation of capital.

16 For particular values of the parameters entering the utility function and the age-earnings profile, the two alternative tax systems may yield model solutions with rates of return on capital outside the range determined by the gross and net of tax rates of return of the income tax system. For instance, the wage tax alternative may yield a rate of return on capital higher than the gross of tax rate of return of the income tax system, while the consumption tax alternative may produce a rate of return lower than the net of tax rate of return of the income tax system.

17 The parameter values of the utility function resulting from the calibration exercise in Part II rule out the case for indebtedness in the early years of economic life.

18 Summers [1981a] offers two reasons for explaining the welfare superiority of a consumption tax over a wage tax. In his words (p. 542), "capital intensity is higher with consumption taxes, which raises welfare by making possible a higher level of steady-state consumption. The second reason is probably more important. Consumption taxation reduces the present value of the taxes each individual pays during his lifetime." The latter point is not necessarily true in a general equilibrium context. While consumption taxation extracts revenue later in life than does wage taxation, the present value of consumption taxes is calculated using a lower equilibrium rate of return as the discount factor. Results in Table 3 yield higher present values of consumption taxes over wage taxes. In the Cobb-Douglas case, the present value of the wage tax is 7.11 and the present value of the consumption tax is 7.55. The respective figures for the case where $\sigma = .5$ are 15.26 and 18.11.

19 This figure is in the same order of magnitude as the aggregate effective marginal rates of capital income taxation estimated by Daly, Jung, Mercier, and Schweitzer [1985].

20 For the case with Cobb-Douglas production, expression (46) for the optimal consumption tax can be written as:

$$t_c = 1 + \left[\frac{(D+1)V}{D} - \frac{1}{F} + \frac{\psi}{\Psi}\right]\left[\frac{\Pi}{L}\left(\frac{n+g}{\alpha}\right)^{\frac{1}{1-\alpha}} - \frac{(1-\alpha)}{\alpha}(n+g)\right].$$

21 The neutrality of the consumption tax with respect to savings can be ascertained by observing that t_c does not appear in expression (32) for aggregate saving.

22 Compared to the figures of note 18 above, the present value of the optimal taxes are 11.78 in the Cobb-Douglas case, and 22.42 when $\sigma=.5$.

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